STUDY AID 5

Dimensional Analysis and Stoichiometry

Chemistry is a quantitative science and involves measurements, complex calculations, and problem solving. Dimensional analysis is an important tool of chemistry, just like the electronic calculator, and measuring devices (like the thermometer, buret, and graduated cylinder).

In dimensional analysis the units for all quantities are always carried along with their corresponding number, the units for the answer come out of the calculations automatically, and errors in the reasoning behind a series of calculations are easily identified and corrected.

Dimensional analysis as a problem-solving tool has many applications in chemistry and is used in many of the laboratory experiments in this manual including: the conversion of one unit into another, calorimetry, solution concentrations, moles and stoichiometry, gas laws, and heat of reaction. Regardless of the application, the basis of dimensional analysis is the use of conversion factors to organize a series of steps in quest of a specific quantity with a specific unit.

A. Conversion Factors

Conversion factors come from equivalent relationships or ratios between two quantities. These relationships are usually expressed as equations or derived units. When used as conversion factors, they are written in fractional form. Some specific examples are shown below:

Example Equivalence Statement or Derived Unit	Conversion Factor #1	Conversion Factor #2
1 mole H_2O = 1 molar mass H_2O	$\frac{1 \text{ mole } H_2O}{18.01 \text{ g } H_2O}$	$\frac{18.01 \text{ g } H_2O}{1 \text{ mole } H_2O}$
1 atmosphere = 760 mm Hg	$\frac{1 \text{ atm}}{760 \text{ mm Hg}}$	$\frac{760 \text{ mm Hg}}{1 \text{ atm}}$
4.184 J/g°C (Specific heat of water)	$\frac{4.184 \text{ J}}{\text{g°C}}$	$\frac{1 \text{ g°C}}{4.184 \text{ J}}$
8.96 g Cu/1 mL (density of Cu)	$\frac{8.96 \text{ g Cu}}{1 \text{ mL}}$	$\frac{1 \text{ mL}}{8.96 \text{ g Cu}}$
22.4 L = 1 mol gas at STP	$\frac{22.4 \text{ L}}{1 \text{ mol gas}}$	$\frac{1 \text{ mol gas}}{22.4 \text{ L}}$

B. Unit Conversions

The dimensional analysis method of converting units involves organizing one or more conversion factors into a logical series that cancels or eliminates all units except the unit(s) wanted in the answer

Problem 1. What is the volume of a 0.15 lb sample of Cu?

The calculation setup requires two conversion factors: lb $\longrightarrow$ g $\longrightarrow$ mL

$$(0.15\ \text{lb Cu})\left(\frac{453.6\ \text{g}}{1\ \text{lb}}\right)\left(\frac{1\ \text{mL}}{8.96\ \text{g Cu}}\right) = 7.6\ \text{mL Cu}$$

Note, that in completing this calculation, units are treated as numbers, lb in the denominator are canceled into lb in the numerator and g in the denominator are canceled into g in the numerator.

Problem 2. How many grams of sodium chloride are in 0.250 L of a solution with a density of 1.04 g/mL that is 10.0% sodium chloride?

The calculation setup requires three conversion factors:

$$\text{L} \longrightarrow \text{mL} \longrightarrow \text{g NaCl}(aq) \longrightarrow \text{g NaCl}$$

$$(0.250\ \text{L NaCl}(aq))\left(\frac{1000\ \text{mL}}{1\ \text{L}}\right)\left(\frac{1.04\ \text{g}}{1\ \text{mL}}\right)\left(\frac{10.0\ \text{g NaCl}}{100.0\ \text{g NaCl}(aq)}\right) = 26.0\ \text{g NaCl}$$

C. The Mole and Stoichiometry

It is often necessary to calculate the amount of product that can be obtained from a given amount of reactant or, conversely, to determine how much reactant is required to produce a stated amount of product. Calculations of this kind, based on balanced chemical equations, are called **stoichiometry** (from Greek, meaning element measure).

In solving stoichiometric problems, conversion factors based on **the mole** are very important. In its broadest sense a mole is Avogadro's number (6.022×10^{23}) of any chemical species. Even though the unit "mole" is used as a short expression for molar mass, it is quite permissible to refer to moles of chemical species that are not really molecular in character. Reference may be made to moles of such diverse species as sulfur atoms (S), oxygen atoms (O), oxygen molecules (O_2), sulfuric acid molecules (H_2SO_4), sodium chloride formula units (NaCl), ammonium ions (NH_4^+), nitrate ions (NO_3^-), or even to moles of electrons or protons.

From these definitions of mole we can write two equivalence statements on which to base conversion factors.

1 mole = 6.022×10^{23} items

1 mole = molar mass in g (mass in grams numerically equal to the molar mass)

Three more conversion factors based on the mole are also useful. One applies only to gases: 1 mole = 22.4 L of gas at STP. The rationale for this is based on Avogadro's famous hypothesis that "equal volumes of all gases, at the same temperature and pressure, contain the same number of molecules". The second applies to solutions for which the concentration is expressed as **molarity**. Molarity is defined as the number of moles of solute in 1 L of solution. The last is based on the mole ratios of reactants and products in a balanced equation. For example, in the hypothetical reaction

$$2A + 3B \longrightarrow A_2B_3$$

the mole ratios of the reactants and product to each other are 2 mol A to 3 mol B to 1 mol A_2B_3. The following table includes the basic equivalence statements or derived units which include the mole.

Equivalence Statement	Conversion Factor #1	Conversion Factor #2
1 mol = 6.022×10^{23} items	$\frac{1\text{ mol}}{6.022 \times 10^{23}\text{ items}}$	$\frac{6.022 \times 10^{23}\text{ items}}{1\text{ mol}}$
1 mol = molar mass in g	$\frac{1\text{ mol}}{\text{molar mass in g}}$	$\frac{\text{molar mass in g}}{1\text{ mol}}$
1 mole = 22.4 L of gas at STP	$\frac{1\text{ mol}}{22.4\text{ L at STP}}$	$\frac{22.4\text{ L at STP}}{1\text{ mol}}$
moles solute/1 L solution	$\frac{\text{moles solute}}{1\text{ L of solution}}$	$\frac{1\text{ L solution}}{\text{moles solute}}$
moles species A/moles species B	$\frac{\text{moles A}}{\text{moles B}}$	$\frac{\text{moles B}}{\text{moles A}}$

Using dimensional analysis involving mole conversion factors to solve a problem requires four steps:

1. Write the balanced chemical equation for the reaction involved.
2. Examine the problem statement and determine what is the given substance that will be the starting point for the calculation.
3. Set up a series of conversion factors that eliminate (by cancellation) all units except the unit specified for the answer.
4. Do the calculations and express the answer with the correct number of significant figures.

Apply these steps to the following problems related to the chemical decomposition of potassium chlorate to produce potassium chloride and oxygen.

Problem 3. How many grams of oxygen can be obtained from 15.0 g of $KClO_3$?

1. Balanced equation: $2\,KClO_3 \longrightarrow 2\,KCl + 3\,O_2$
2. The problem states that 15.0 g of $KClO_3$ is being converted to O_2. Therefore, the dimensional analysis setup begins with 15.0 g $KClO_3$

3. The calculation setup will be determined by choosing conversion factors that cancel units in the preceding quantity or conversion factor. For this problem, three conversion factors are needed:

$$\text{g KClO}_3 \longrightarrow \text{mol KClO}_3 \longrightarrow \text{mol O}_2 \longrightarrow \text{g O}_2$$

$$(15.0\ \cancel{\text{g KClO}_3})\left(\frac{1\ \cancel{\text{mol KClO}_3}}{122.6\ \cancel{\text{g KClO}_3}}\right)\left(\frac{3\ \cancel{\text{mol O}_2}}{2\ \cancel{\text{mol KClO}_3}}\right)\left(\frac{32.00\ \text{g O}_2}{1\ \cancel{\text{mol O}_2}}\right)$$

(Note: 122.6 g and 32.00 g are the molar masses of $KClO_3$ and O_2, respectively).

4. Once all the units have been canceled except the units specified for the answer, the calculations can be completed.

$$\frac{(15.0)(3)(32.00)\ \text{g O}_2}{(122.6)(2)} = \mathbf{5.87\ g\ O_2}$$

Problem 4. How many oxygen molecules can be obtained from 15.0 g of $KClO_3$?

Steps 1, 2 and 4 are exactly the same as in Problem 3, but the last conversion in step 3 requires a different conversion factor:

$$\text{g KClO}_3 \longrightarrow \text{mol KClO}_3 \longrightarrow \text{mol O}_2 \longrightarrow \text{molecules O}_2$$

$$(15.0\ \cancel{\text{g KClO}_3})\left(\frac{1\ \cancel{\text{mol KClO}_3}}{122.6\ \cancel{\text{g KClO}_3}}\right)\left(\frac{3\ \cancel{\text{mol O}_2}}{2\ \cancel{\text{mol KClO}_3}}\right)\left(\frac{6.022 \times 10^{23}\ \text{molecules O}_2}{1\ \cancel{\text{mol O}_2}}\right)$$

$$= \mathbf{1.11 \times 10^{23}\ molecules\ O_2}$$

Problem 5. How many liters of oxygen gas, measured at STP, can be obtained from 15.0 g of $KClO_3$?

Steps 1, 2, and 4 are exactly the same as in Problems 3 and 4, but the last conversion in step 3 requires a different conversion factor:

$$\text{g KClO}_3 \longrightarrow \text{mol KClO}_3 \longrightarrow \text{mol O}_2 \longrightarrow \text{L O}_2$$

$$(15.0\ \cancel{\text{g KClO}_3})\left(\frac{1\ \cancel{\text{mol KClO}_3}}{122.6\ \cancel{\text{g KClO}_3}}\right)\left(\frac{3\ \cancel{\text{mol O}_2}}{2\ \text{mol KClO}_3}\right)\left(\frac{22.4\ \text{L}}{1\ \text{mol O}_2}\right) = \mathbf{4.11\ L}$$

Problem 6. How many grams of $KClO_3$ must be decomposed to produce 25.0 g of KCl?

Step 1. Balance the equation for the reaction as shown in Problem 1.

Step 2. Consider KCl to be the given substance; convert to g $KClO_3$ using a sequence of conversion factors.

$$\text{g KCl} \longrightarrow \text{mol KCl} \longrightarrow \text{mol KClO}_3 \longrightarrow \text{g KClO}_3$$

Steps 3 and 4. Choose conversion factors to cancel units of the preceding fractions in the sequence.

$$(25.0 \text{ g KCl})\left(\frac{1 \text{ mol KCl}}{74.55 \text{ g KCl}}\right)\left(\frac{2 \text{ mol KClO}_3}{2 \text{ mol KCl}}\right)\left(\frac{122.6 \text{ g KClO}_3}{1 \text{ mol KClO}_3}\right) = \mathbf{41.1\ g\ KClO_3}$$

Problem 7. How many mL of 6.0 M HCl(*aq*) are needed to react with 4.85 g of $NaHCO_3$?

Step 1. The balanced equation for this reaction is

$$NaHCO_3 + HCl \longrightarrow NaCl + H_2O + CO_2$$

Step 2. Consider $NaHCO_3$ to be the given substance; convert to mL of 6.0 M HCl using a sequence of conversion factors.

$$\text{g NaHCO}_3 \longrightarrow \text{mol NaHCO}_3 \longrightarrow \text{mol HCl} \longrightarrow \text{L HCl} \longrightarrow \text{mL HCl}$$

Steps 3 and 4. Choose conversion factors to cancel units of the preceding factors in the sequence.

$$(4.85 \cancel{\text{g NaHCO}_3})\left(\frac{1 \cancel{\text{mol NaHCO}_3}}{84.01 \cancel{\text{g NaHCO}_3}}\right)\left(\frac{1 \cancel{\text{mol HCl}}}{1 \cancel{\text{mol NaHCO}_3}}\right)\left(\frac{\cancel{1 \text{L}}}{6.0 \cancel{\text{mol HCl}}}\right)\left(\frac{1000 \text{ mL}}{\cancel{1 \text{L}}}\right)$$

$$= \mathbf{9.6\ mL\ HCl}$$